Reprinted from THE JOURNAL OF CHEMICAL PHYSICS, Vol. 23, No. 7, 1268–1273, July, 1955
Printed in U. S. A.

Measurement of the Chapman-Jouguet Pressure and Reaction Zone Length in a Detonating High Explosive

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(Received September 13, 1954)

The Chapman-Jouguet pressure and the reaction zone length in detonating Composition B containing 63 percent RDX at a density of 1.67 g/cc have been measured by determining the initial free surface velocity imparted to aluminum plates as a function of plate thickness. The *C-J* pressure is 0.272 megabar and the reaction zone length is 0.13 mm. The experimental free surface velocity-plate thickness curve provides powerful confirmation for the pressure profile in a detonating explosive predicted by the hydrodynamic theory of detonation proposed by Zeldovich, von Neumann, and Doring.

INTRODUCTION

IN 1945 Goranson[1] suggested that the reaction zone of a detonating solid explosive could be investigated by determining the initial free surface velocity imparted to thin metal plates as a function of plate thickness. In particular he showed that reaction zone length and Chapman-Jouguet pressure could be estimated in this way. Unfortunately, the original results obtained by Goranson are available only in a classified report from this laboratory. This paper describes similar but improved theoretical and experimental results obtained during and since 1950.

THEORY OF THE EXPERIMENT

The generally accepted picture of the structure of a steady-state, plane detonation wave was proposed independently by Zeldovich,[2] von Neumann,[3] and Döring.[4] The wave is assumed to consist of a nonreactive shock followed by a steady-state reaction zone which is terminated at the Chapman-Jouguet surface where the local flow velocity plus sound speed equals the detonation velocity. It can be shown that this condition is fulfilled at the point of tangency in the p-v plane of a straight line from the initial state to the final Hugoniot curve calculated for a fixed composition of the product gases.[5] Figure 1 shows a representation of the detonation process in the p-v plane. Reference to the laws of conservation of mass and momentum shows immediately that if viscosity and heat conduction are neglected, the succession of state points assumed by the reacting explosive is represented by the straight line from p_1v_1 to the *C-J* point. The rate at which an element of explosive passes from p_1v_1 to the *C-J* state depends on the kinetics of the reactions involved and cannot be determined from hydrodynamic considerations. It follows, therefore, that the pressure-distance profile of a detonation wave consists most probably of a monotone but otherwise unspecified decrease in pressure from p_1 to $p_{C\text{-}J}$ in an unspecified distance. Two profiles often considered are shown qualitatively in Fig. 2. They correspond to a reaction rate determined by grain burning and to a rate determined by a first or second order adiabatic reaction of the Arrhenius type. The unsteady flow behind the Chapman-Jouguet plane has been investigated by Taylor.[6] This flow may be simply described as a rarefaction wave ending either in cavitation or in a steady-state region required to match

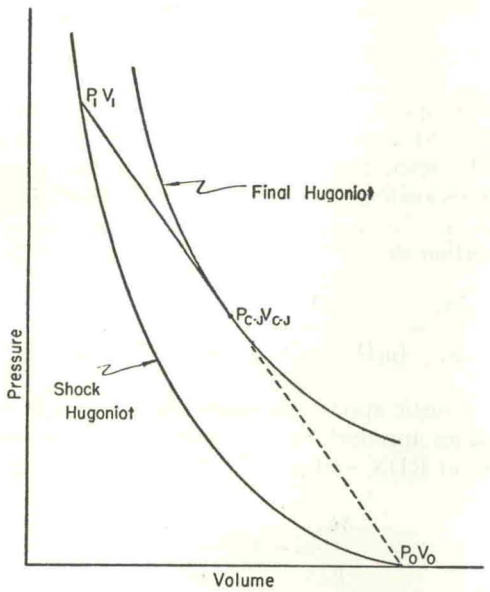

FIG. 1. A representation of the detonation process in the pressure-volume plane.

[1] R. W. Goranson, Classified Los Alamos Report LA-487.
[2] Y. B. Zeldovich, J. Exptl. Theoret. Phys. (USSR) **10**, 542 (1940).
[3] J. von Neumann, O.S.R.D. Report No. 549, (1942).
[4] W. Döring, Ann. Physik **43**, 421 (1943).
[5] More specifically, the *C-J* point is the tangent point on a Hugoniot curve for the product gases whose composition is assumed fixed at the equilibrium values appropriate for the tangent point. The relation of this statement of the Chapman-Jouguet condition to the usual statement, namely, that the *C-J* point is the tangent point on a Hugoniot every point of which is in chemical equilibrium, is not clear at the moment because of uncertainties in the equation of state of the detonation products. The former statement can be derived from a recent theoretical investigation of the structure of a steady-state plane detonation wave by Kirkwood and Wood, J. Chem. Phys. **22**, 1920 (1954). It should be mentioned that this statement of the *C-J* condition has been shown to apply to all detonations which are not pathological in the von Neumann sense. Finally, no pathological detonation has yet been observed.
[6] G. I. Taylor, Proc. Roy. Soc. (London) **A200**, 235 (1950).

boundary conditions at the back boundary of the explosive products.

When a plane detonation wave is incident normally on a metal plate, a shock wave is transmitted into the metal which is followed by a rarefaction wave corresponding to the pressure drop in the reaction zone of the explosive. The foot of this rarefaction wave will travel with a velocity equal to the sum of the local flow velocity and sound speed. It will overtake the shock in the metal after the shock has been attenuated by the rest of the rarefaction wave. The strength of the shock wave will decrease relatively quickly as this interaction proceeds because of the small thickness of the reaction zone. As a result, the velocity imparted to a thin metal plate, which depends directly on the strength of the shock in the plate, should change with plate thickness qualitatively as shown in Fig. 3.

It has been shown that to a very good approximation the shock particle velocity of a metal in the high

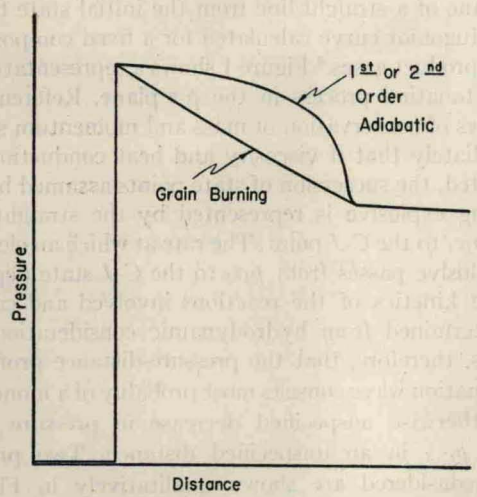

FIG. 2. Two representative pressure profiles for the reaction zone of solid explosives.

explosive pressure range is one-half of the free surface velocity.[7] This fact makes it possible to determine the Chapman-Jouguet pressure in the explosive from the free surface velocity of a metal plate corresponding to the end of the interaction caused by the reaction zone. This velocity is v in Fig. 3. An immediate consequence of the laws of conservation of mass and momentum is that the pressure behind a shock wave moving into a medium at rest is

$$p=\rho_0 u D, \tag{1}$$

where ρ_0 is the density of the unshocked material, u is the shock particle velocity, and D is the shock velocity. In all of the experiments discussed in the next section the metal used was aluminum or dural. For these materials Walsh[7] has determined experimentally a relation between shock velocity and free surface velocity

[7] J. M. Walsh and R. H. Christian, Phys. Rev. **97**, 1544 (1955).

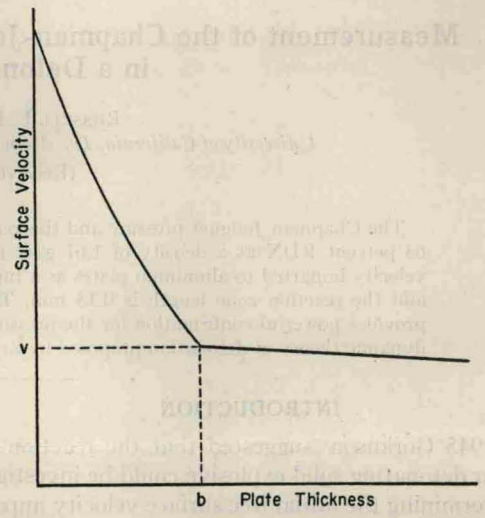

FIG. 3. Free surface velocity of a metal plate as a function of plate thickness showing the high velocity produced in thin plates by the von Neumann spike in the explosive.

in an investigation of the equation of state of the metals. The pressure in the metal can therefore be determined from the measured metal density, the free surface velocity, and this relation.

By applying the usual boundary conditions of equality of pressure and continuity of flow velocity at the interface between explosive and metal, the following expression can be developed relating incident pressure in the explosive to transmitted pressure in the metal:

$$\frac{p_m}{p_x}=\frac{\rho_2 D_2}{\rho_1 D_1}\left(\frac{\rho_1 D_1+\rho_3 D_3}{\rho_2 D_2+\rho_3 D_3}\right). \tag{2}$$

The subscripts 1, 2, and 3 refer to the properties of the undetonated explosive, the metal, and the explosive products, respectively. The pressure in the explosive, p_x, corresponding to the Chapman-Jouguet state can now be calculated if $\rho_3 D_3$ is known. An error analysis of this relation shows that

$$\frac{\delta p_x}{p_x}=\frac{(\rho_1 D_1-\rho_2 D_2)\rho_3 D_3}{(\rho_1 D_1+\rho_3 D_3)(\rho_2 D_2+\rho_3 D_3)}\frac{\delta(\rho_3 D_3)}{\rho_3 D_3}.$$

If the acoustic approximation is made that $\rho_1 D_1=\rho_3 D_3$ and values appropriate for Composition B (nominally 60 percent RDX−40 percent TNT) are inserted, then

$$\frac{\delta p_x}{p_x}=-0.1\frac{\delta(\rho_3 D_3)}{\rho_3 D_3}.$$

Thus it is clear that the Chapman-Jouguet pressure is quite insensitive to the value of $\rho_3 D_3$ assumed. Furthermore, the acoustic approximation is correct insofar as the velocity of the reflected shock can be assumed equal to the velocity of a rarefaction wave in the product

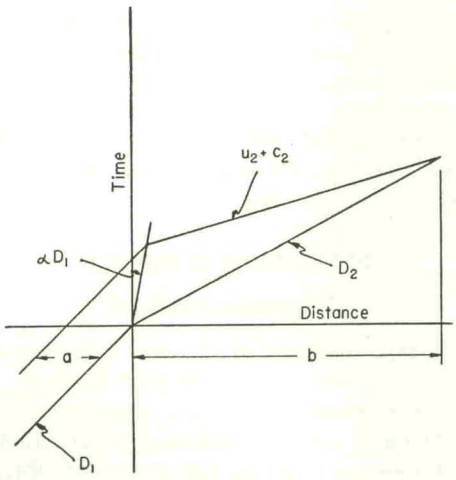

FIG. 4. Distance-time representation of the interactions between the von Neumann spike in the explosive and the metal plate.

gases. Therefore, to a good approximation

$$p_m/p_x=2\rho_2D_2/(\rho_1D_1+\rho_2D_2). \quad (3)$$

An estimate of the reaction zone length can be made from a determination of the distance required for the end of the rarefaction corresponding to the Chapman-Jouguet plane to overtake the shock wave in the metal. This distance is b in Fig. 3. If it is assumed that the metal is a perfect impedance match to the explosive so that no wave is reflected back into the explosive, an $x-t$ representation of the interaction will be as shown in Fig. 4. The interface is initially assumed to be at $x=0$. A detonation wave comes in from the left with a velocity D_1. The reaction zone length is a. The velocity of the interface through the reaction zone is αD_1, the shock velocity in the metal is D_2, and the velocity of the foot of the rarefaction wave is u_2+c_2. D_2 and α will vary as the interaction proceeds. The values used in the formula below and those indicated in Fig. 4 are the appropriate average values. As long as the flow behind the shock can be considered isentropic (a good approximation), u_2+c_2 depends only on pressure and is the value corresponding to the C-J state transmitted into the metal. Simple analytical geometry leads to the following relation between the reaction zone length in the explosive and the interaction distance in the metal:

$$a=b[D_1(u_2+c_2-D_2)(1-\alpha)][D_2(u_2+c_2-\alpha D_1)]^{-1}. \quad (4)$$

The appropriate average value of D_2 can be determined from the free surface velocity-plate thickness relation since at every value of thickness this relation gives the corresponding shock velocity by using the equation of state. It can easily be seen that the appropriate average to be used in the above equation is the inverse average; i.e.,

$$D_2=\left(\frac{1}{b}\int_0^b \frac{dx}{D(x)}\right)^{-1}. \quad (5)$$

Likewise α should be determined from a similar inverse average of the interface velocity.

It is possible to calculate in detail the free surface velocity as a function of plate thickness by using the procedure outlined by Courant and Friedrichs[8] under the assumption mentioned above; namely, that the flow behind the shock can be considered isentropic. In particular, if the reaction zone rarefaction and the Taylor wave are both assumed to be centered rarefaction waves, and if a ratio of explosive charge length and reaction zone length is assumed, the ratio of slopes of the free surface velocity-thickness curve for the two waves at the point corresponding to the end of the reaction zone can be calculated. This ratio is about 25 if the charge length is 200 times the reaction zone length and if the spike pressure is 1.5 times the Chapman-Jouguet pressure. In practical cases the ratio of charge length to reaction zone length is much greater than 200 so the change of slope would be even larger than indicated above. This calculation justifies the sharp change in slope of the free surface velocity-plate thickness curve shown in Fig. 3.

It should be remembered that it was assumed in this calculation that the reaction zone rarefaction could be approximated by a centered rarefaction wave. However, most reaction rate expressions show that equilibrium is approached asymptotically in time. If such expressions are appropriate for solid explosives, and there is no experimental evidence that they are, the ratio of slopes could conceivably approach unity. However, the long reaction tail predicted by these kinetic expressions corresponds to a very small percentage of the total detonation reaction, and a rapid rate of change of slope would be expected not at a plate thickness corresponding to the C-J state but at a thinner one, corresponding to essentially complete reaction.

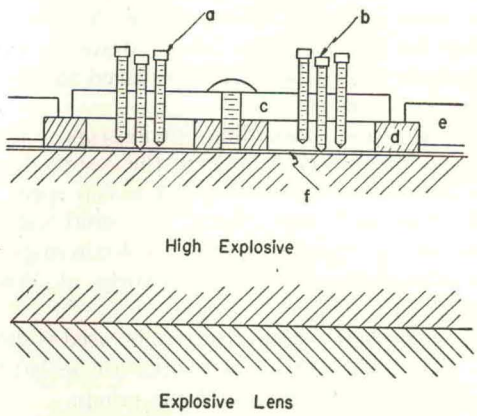

FIG. 5. Cross section of an experiment to determine the free surface velocity of a very thin plate or foil. (a) 0–80 pointed steel screws used as surface velocity pins. (b) Center ground contact pin also used to hold foil against the high explosive. (c) Texalite pin mounting plate. (d) Aluminum backing plate $3\frac{1}{4}$ in. o.d., $\frac{1}{4}$ in. thick. (e) Blast shield 8 in. o.d., $\frac{3}{8}$ in. thick. (f) Aluminum foil.

[8] R. Courant and K. O. Friedrichs, *Supersonic Flow and Shock Waves* (Interscience Publishers, Inc., New York, 1948), p. 164.

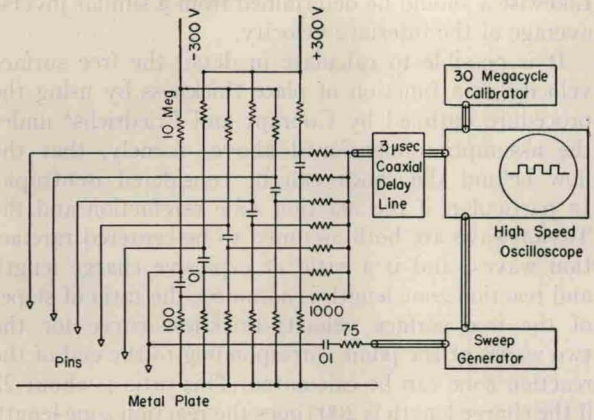

FIG. 6. Electronic circuitry used to measure free surface velocity. The component values are the same for all circuits. Resistance values are ohms and capacitor values μfd unless otherwise indicated.

EXPERIMENTAL TECHNIQUE

Free surface velocity was measured by recording the time of arrival of the metal surface at a series of metal contactors or pins. The technique used is identical with that described by Minshall.[9] In these experiments two or in some cases four velocity measurements were made in each experiment by using either two or four groups of pins each consisting of six pins set in a $\frac{1}{4}$-in. diameter circle. The spacing between pins measured perpendicular to the free surface was 0.003 in. for the thin plates 0.030 in. thick and under, and 0.006 in. for all thicker plates. A cross section through a thin foil experiment is shown in Fig. 5.

The electronic circuitry used is indicated in Fig. 6. As each pin is shorted to the moving plate, a condenser is discharged through the network producing a signal on the oscilloscope. Successive pins have alternate polarities so that an ideal record would resemble the square wave indicated in the figure. A time base is provided by a trace from an accurately calibrated 30-megacycle crystal oscillator displaced slightly from the velocity record.

The high-speed recording oscilloscopes used in these experiments were similar to the system described by the Radiation Laboratory.[10] A sweep speed of 10 in./μsec was used and twenty thousand volts post acceleration was required in order to obtain single traces intense enough to photograph clearly at this high writing speed.

A photograph of a setup ready to be used is shown in Fig. 7. The pulse-forming networks are sealed in the plastic blocks located near each pin group.

Precautions were necessary to eliminate two possible sources of error. The shock wave in air preceding the free surface was strong enough to discharge the pin circuits before the arrival of the surface. This difficulty was eliminated by maintaining a methane atmosphere around the pins. If velocity measurements were made on a machined and polished surface, the first several pins were often discharged prematurely by what appeared to be a fine spray of metal jetted out from the surface. This spray was eliminated by using unworked surfaces whenever possible and covering the surface with a very thin, almost invisible coating of light oil.

EXPERIMENTAL RESULTS

Experimental Data

Thirty-three separate experiments were performed in which eighty-seven measurements of free surface velocity were made. In the first twelve of the experiments the metal used was dural and the remainder pure aluminum was used. Walsh[7] has shown that the particle velocity-shock velocity relationship is identical within experimental error for both metals. The explosive was Composition B which was cast into large blocks and then machined into pieces $5\frac{1}{2}$ in.$\times 5\frac{1}{2}$ in.$\times 3$ in. The composition and density of the explosive varied by roughly 2 percent RDX and 0.01 g/cc from charge to charge and within a given charge. These charges were prepared in 1950 and are not of as high a quality as those prepared currently.

All of the experimental results were corrected to the following standard conditions: metal density$=2.71$ g/cc; explosive density 1.67 g/cc; explosive composition 63 percent RDX; and detonation velocity 7.868 mm/μsec. The following error expression was used in this correction:

$$\delta u_2/u_2 = -0.861\delta\rho_1 - 0.0023\delta \text{ percent RDX} + 0.192\delta\rho_2.$$

In most cases the corrections made velocity changes of less than 1 percent. A random error of $1\frac{1}{2}$ percent is

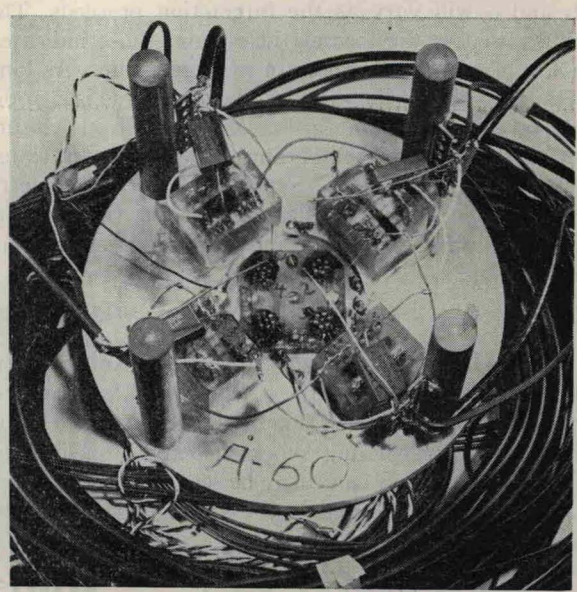

FIG. 7. A photograph of an experiment ready to be fired.

[9] F. S. Minshall, J. Appl. Phys. **26**, 463 (1955).
[10] M.I.T. Radiation Laboratory Report No. 1001 (1946).

caused by the composition and density fluctuations within a given charge. The average values of the velocities measured are presented in Table I. These data are also presented in Fig. 8.

Two subjects deserve comment before conclusions are drawn from this data. First, the standard deviations reported in Table I are unfortunately large. However, this large standard deviation arises mainly from a systematic source. As discussed above, velocities are determined from measurements of position and time of the metal free surface by means of pins set in a circle. If the metal surface is not perfectly plane and if it does not move perfectly parallel to its initial position, a systematic error in the arrival time of the surface at each pin will be introduced which may be reflected into the surface velocity as determined by least squares techniques. It can be shown that the velocity calculated is related to the true velocity as follows:

$$V = V_{\text{calc}}\left(1+\frac{R\alpha}{d}\epsilon\sin\theta\right),$$

where R is the radius of the pin circle, d is the incremental pin spacing, α is the angle of tilt of the surface, ϵ is a constant $=0.11$ for the pin geometry used in these experiments, and θ is an angle which describes the orientation of the pin circle with respect to the tilted wave. A wave tilt of as much as 0.03 radian was observed for some of the charges used. Therefore, this cause alone could produce a velocity error of 15 percent in plates 0.030 in. thick or thinner and $7\frac{1}{2}$ percent for thicker plates. For this reason many measurements were made, especially on the thinner plates, so as to obtain a reliable value for the average velocity.

Second, it is desirable to place all of the pins close enough to the free surface so that the velocity measurement can be completed before a second disturbance arrives at the surface. This was done for all but the thinnest foil. One might expect a weak shock wave to be the second disturbance to arrive at the surface giving it a small increase in velocity at about the middle of the velocity measurement. Examination of the records does not indicate a noticeable increase in velocity. However, the velocity associated with the 0.0085 in. foil may be slightly high.

Explosive Pressure

Two pressures in the explosive can be estimated from the surface velocity plot of Fig. 8. A least squares straight line has been fitted to the experimental measurements at thicknesses greater than 0.030 in. Each of the average velocities was given a weight equal to the number of measurements included. A smooth curve was drawn through the remaining four measurements on thin plates. The Chapman-Jouguet pressure can be determined from the free surface velocity

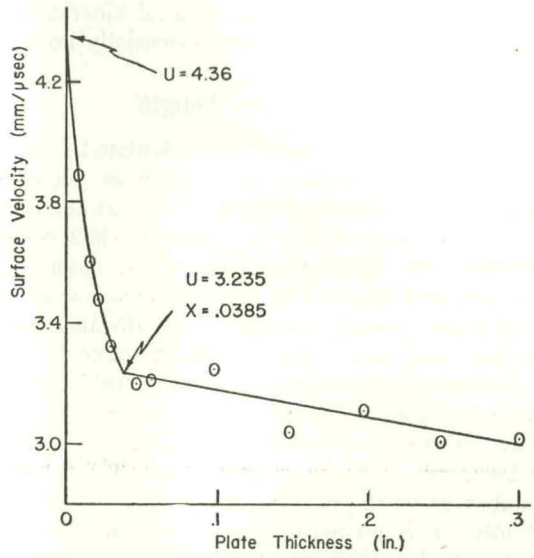

FIG. 8. Measured free surface velocity as a function of plate thickness.

indicated by the intersection of these two lines, 3.235 mm/μsec.

In the metal[11]

$$p_m=\rho_2 u_2 D_2$$

$$=2.71\times\frac{3.235}{2}\times 7.55=0.3309 \text{ megabar.}$$

In the explosive

$$p_{C\text{-}J}=\frac{p_m(\rho_1 D_1+\rho_2 D_2)}{\rho_2 D_2}=0.272 \text{ megabar.}$$

Thus the Chapman-Jouguet pressure in Composition B explosive containing 63 percent RDX at a density of 1.67 g/cc is 0.272 megabar. This number is thought to be correct to within 2 percent.

The extrapolation of the free surface velocity to zero plate thickness in what is thought to be a reasonable manner gives a limiting velocity of 4.36 mm/μsec. From this number and the extrapolation of the equation of state data for aluminum made by Fickett, a peak pressure in the explosive of 0.385 megabar is estimated for the von Neumann spike. It is interesting to note that the spike pressure appears to be only 1.42 the Chapman-Jouguet pressure. It should be emphasized that the extrapolation to zero thickness is only what appears to be a reasonable one. There is no theoretical justification for the assumed form of the curve because the form

[11] The equation of state data of Walsh, see reference 7, has been analyzed by Fickett (unpublished communication). An analytic form of the equation of state was derived which agreed with Walsh's data at low pressures and with Fermi-Thomas-Dirac calculations at high pressures. The following fit of shock velocity as a function of free surface velocity is appropriate for the pressure range of interest in these experiments:

$$D=4.8375+1.1235u-0.1095u^2+0.0066u^3.$$

depends on the details of the chemical kinetics of the detonation reaction about which essentially nothing is known.

Reaction Zone Length

The reaction zone length was calculated from Eq. (4) using a value of D_2 determined from the experimental results as required by Eq. (5). α was determined from the two values of interface velocity which could be estimated from the experimental data, namely, the initial and final values. The assumption was made that the interface velocity changed with distance in the same way the shock velocity did. A value of u_2+c_2 was obtained from the equation of state calculation for aluminum made by Fickett. The actual numbers used are as follows: $D_1=7.868$, $D_2=7.771$, $u_2+c_2=9.065$ mm/μsec, and $\alpha=0.232$; giving $a/b=0.139$. b was estimated to be 0.0385 in. Therefore, $a=0.005$ in. or 0.13 mm. This estimate of reaction zone length of slightly greater than one-tenth of a millimeter is probably accurate to within 20 percent except for the possible errors discussed below.

Two assumptions have been made in the estimation of reaction zone length. First the shock wave reflected from the metal back into the explosive has been ignored. This assumption is questionable because the changes in temperature and pressure caused by the wave may decisively influence the kinetics in the as yet unreacted explosive into which it moves. Therefore, the value of reaction zone length determined is probably best described as a lower limit value.

The effect of this reflected shock wave on the detonation kinetics and reaction zone length could be investigated by varying the metal used in experiments of this type. In particular, the effect could be maximized by using a heavy material like brass which has a large acoustic impedance and minimized by using magnesium which is almost a perfect impedance match for Composition B.

The second assumption concerns the shape of the reaction zone. The experimental results have been represented by a profile similar to that of a rarefaction wave in an inert material. However, as discussed above, there could be a slow reaction tail which would cause true reaction zone length to be somewhat longer than that indicated.

TABLE I. Measured free surface velocity as a function of plate thickness.

Plate thickness (in.)	Average velocity (mm/μsec)	Standard deviation of the mean (mm/μsec)	Number of measurements
0.0085	3.89	0.34	12
0.016	3.60	0.26	15
0.021	3.48	0.25	11
0.030	3.32	0.10	11
0.048	3.20	0.20	10
0.057	3.22	0.02	2
0.098	3.25	0.09	8
0.150	3.04	0.06	4
0.198	3.11	0.07	6
0.248	3.01	0.09	4
0.300	3.02	0.04	4

CONCLUSIONS

A conclusion can be drawn from the data presented above which is fundamental to the understanding of the detonation phenomenon. Namely, the experimental results provide powerful confirmation for the hydrodynamic theory of the detonation process proposed by Zeldovich, von Neumann, and Döring. In fact, this is thought to be the first experimental evidence published which directly verifies this theory which has, however, attained almost universal acceptance because of its hydrodynamic completeness.

The Chapman-Jouguet pressure in Composition B explosive containing 63 percent RDX at a density of 1.67 g/cc was measured to be 0.272 megabar. The reaction zone length for the same explosive is 0.13 mm.

ACKNOWLEDGMENTS

The authors are indebted to many people for assistance in various phases of this work. In particular, we are happy to acknowledge the help of the following individuals: R. Holbrook, E. Gittings, R. E. Faudree, and V. Thurlow for assistance in the actual performance of the experiments; C. L. Miller for the preparation of the many explosive charges used; C. W. Mautz for the derivation of Eq. (4); and W. W. Wood for many valuable discussions of the theoretical problems involved and also for making the detailed calculation of the attenuation of a shock wave in metal by a centered simple rarefaction wave.

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